MathsHUBS White Rose

Problem Solving In Japan

In June 2016, I went to Tokyo with a group of teachers as part of the IMPULS project (International Maths-teacher Professionalization Using Lesson Study). The purpose of the trip was to focus on two things: 'Lesson Study' and problem solving in mathematics. During our stay, we visited five schools and observed seven mathematics lessons, ranging from grades 1-10. I'd previously heard a lot about problem solving in Japan, so it was an exciting opportunity!

One thing that took me by surprise was what they regarded as problems. I had expected to see complex, multi-layered problems with lots of context and information, similar to those 4-6 mark questions that we often see in GCSE Maths papers. But there wasn't any of that! Actually, the questions were quite straightforward. For example, one was essentially 48 divided by 3.

> *"Each problem was pitched just beyond the students' current capabilities."*

Questions seemed to be considered as problems to students if they were unlike things they had solved before. For example, if a class has only ever learned how to calculate areas of shapes that have straight sides, then they'll have a problem on their hands when they are asked to find the area of a circle. In the case of the question involving 48 divided by 3, this was a problem to that particular class because it was the first time that they had to think about a division that was beyond their times table facts.

I'd probably go so far to say that the problem-solving element to these lessons wasn't within the questions themselves, but within the approach that classes took to learning new mathematical content. The common theme was to think, "*When something looks unfamiliar to us, how could we think about it using the things that we already know?*" This seems like an important question for any student to ask themself habitually when faced with unfamiliar situations.

Each problem was pitched just beyond the students' current capabilities. Students would have enough knowledge to be able to relate the problem to things that they already knew, but didn't yet know how to solve it. Therefore they had to either manipulate the problem or manipulate their prior knowledge in a new way solve it. It was through this process that students gained new insights about the topic (e.g. that you can divide large numbers by partitioning the dividend).

I wasn't fully sure if there was a set lesson structure that all schools abided to, but there were aspects that were consistent in all lessons. The main thing that they all had in common was that they spent the majority of lesson time looking at one single problem. They didn't just solve the problem; they solved it several times and discussed it in depth! Most lessons seemed to follow a structure like this:

Starter
Activate prior learning that is related to today's problem.

↓

Introduce the problem
Students attempt the problem for themselves.

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Class discussion
Compare the different methods that pupils have tried.

↓

Reflection
Students write and share what they have learned.

↓

Application
Students use what they have learned to solve other problems.

Here's what happened in each part…

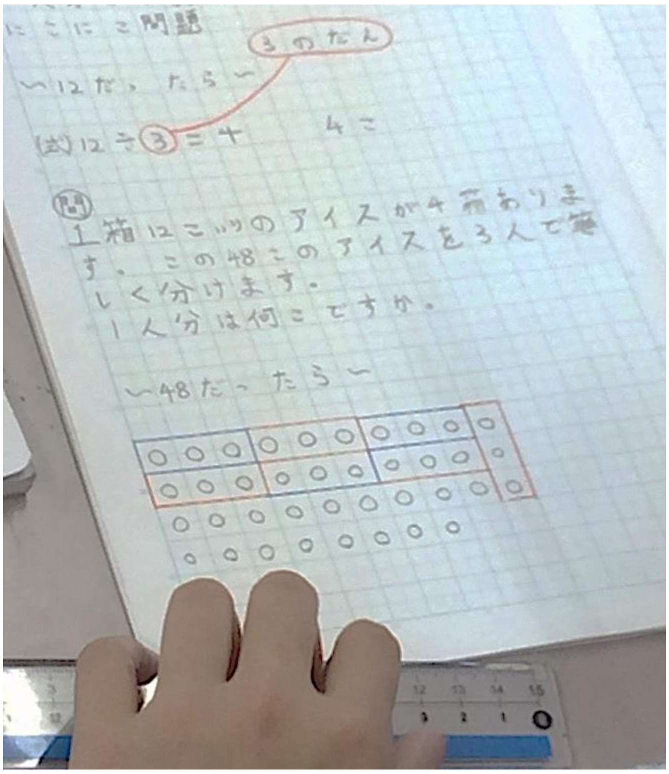
1. Starter: activate prior learning that is related to today's problem.

Each lesson started with the students answering a couple of questions on things that they had previously learned. The questions were far from random. Sometimes they were on what they did last lesson; other times they were on things they'd learned long ago. Either way, the questions would be linked somehow to the problem that they were about to face in the main part of the lesson.

This seemed like a crucial part of the lesson. It was almost like when someone lays out all their tools on the table before starting a piece of DIY so that they are ready to use. These questions reactivated students' prior knowledge and got them thinking about the key pieces of information that were about to need. They were now primed and ready to think about today's problem.

2. Introduce the problem: students attempt the problem for themselves.

There was no set way that teachers did this. Sometimes the students worked independently; sometimes they worked in pairs or groups. In most lessons, students were asked to record their ideas using a combination of pictures, calculations and words.



At first, I mistook this strategy for a 'discovery learning' approach. It certainly looked like that by the way that students were left to navigate their own way through the problem and explore their own methods. However, it was only by looking at the lesson plans that I could see that it wasn't quite as free as that. The students did exactly what the teacher had expected them to.

"Students were given the space to struggle."

The lesson plans included selections of methods the teacher had anticipated that students would use… and they did! Most lesson plans also identified the precise moments when students might have difficulty… and they did! The teachers were able to do this so accurately by using what they knew of the students' prior knowledge and by also considering how the starter activity would prime students for the main problem.

While students were working, the teacher would circulate around the classroom but would rarely help them (if at all). They gave students the space to struggle. So instead of helping, the teacher would read or listen to the students' ideas, make a note of what methods they were trying out and look out for misconceptions. The teacher used this information to plan how the next class discussion would go.

3. Class discussion: compare the different methods that pupils have tried.

During this phase, the class would consider the different methods that had been used. The teacher often had some methods in mind that they wanted to focus on, and sometimes a particular order for them too. So, certain students were picked to describe to the class what they had done and their methods were modelled on the board. While this was happening, the teacher would regularly interject with questions to the rest of the class, such as *"What do these calculations mean so far? What do you think she did next? Talk about why she might have chosen to do this."*

Nothing was skimmed over and the level of detail felt forensic! For example, with a calculation like $3 \times 4 + 5 \times 4$ the teacher might ask, *"What part of the problem does this 3 represent? What does this 4 represent? Here's another 4. Is it the same as the first 4 or does it represent something different? And why are we adding them?"* I've tried this with my own classes and it's made me realise how often students can connect numbers together with correct calculations without fully understanding what they mean.

It is also worth mentioning that barely anything was rubbed off the board during the lessons. The starter questions would be on left side of the board, followed by the problem, followed by the first method, followed by another method, followed by another. And all this remained on the board until the lesson finished. This meant that students could refer back to what had previously been discussed at any point in the lesson. It also allowed the discussions to go beyond a simple 'show and tell' of different methods, as students compared the methods to identify what they all had in common.

4. Reflection: students write and share what they have learned.

The point of exploring multiple methods wasn't to say to the students, *"Find a method you like and use it."* It was to get students to compare methods so that they could deduce some kind of generalisation or conclusion about the topic. For example, in the lesson on 48 divided by 3, each method involved partitioning 48 somehow before dividing it. Therefore, the class concluded that large numbers could be divided by partitioning it into smaller numbers, dividing each part and then adding the answers together – a foundation for learning the bus stop method later down the line.

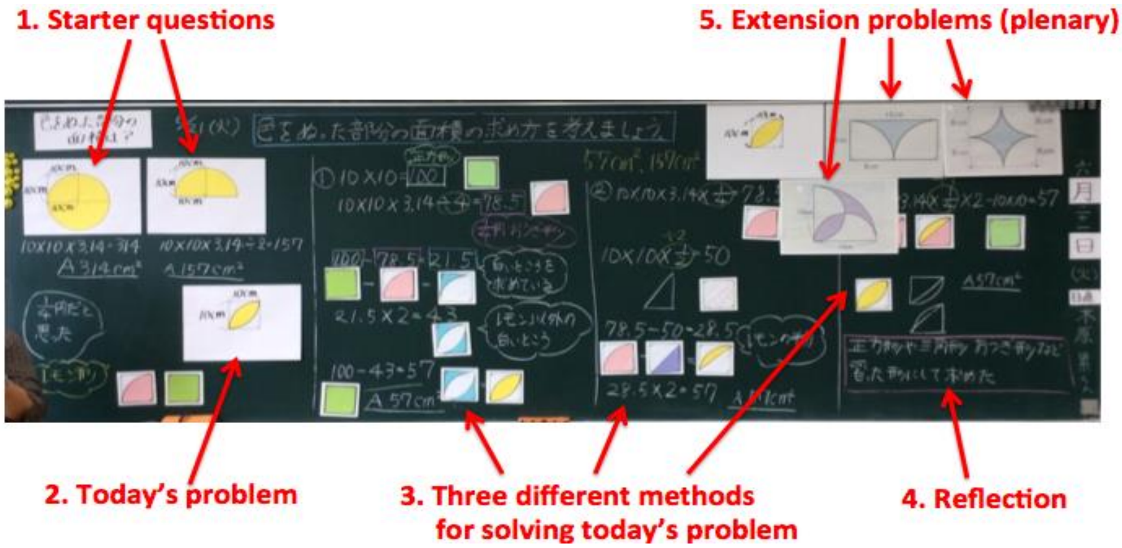
"The level of detail felt forensic!"

Some of these generalisations were drawn out during the class discussions, but students were also asked to write reflections in their books about what they had learned. For example, in a lesson on finding the volume of compound prisms a student wrote, *"You can find the volume of L-shape prisms by changing it into cuboids. You can either split the shape into smaller cuboids and add them together or fill in the empty space and subtract."* (Thanks to the translators!)

Time permitting, the students would then be asked to apply what they had learned by solving similar (but slightly different) problems.

What have we learned from visiting Japan?

Mathematics lessons are not just about teaching students to answer questions; they're about teaching students to think mathematically. The lessons I saw in Japan exemplified this by the sheer depth of class discussions. Students were encouraged to talk analytically about methods and identify the connections between different mathematical ideas. It's hard work for the students, but learning requires hard work!



Want to learn more about exploring and comparing multiple methods? Come to our 'Ratio, Proportion and Rates of Change' course on 28 June or 'Questioning and Variation' on 10 July.